Citation—Joseph Ayoub

Ayoub carried out the program, outlined by Voevodsky, of setting up Grothendieck's six-functor formalism in movitic stable homotopy. That is, he established a formalism of the functors f^*, f_*, $f^!$, $f_!$, $\otimes$ and Hom, and treated the theory of nearby and vanishing cycles, all in the context of the algebraic stable motivic homotopy category. It is a major piece of foundational work, occupying two volumes and more than 800 pages in *Asterisque.* There are also papers where he uses the theory: for example there is a short note where he employs the formalism to give a counterexample to a conjecture of Morel. He begins by producing a singular surface X and an $\mathbb{A}^1$–local complex on X, whose homology sheaves are not $\mathbb{A}^1$–invariant. To deduce the counterexample to Morel's conjecture, which is about smooth X, he then shows how to reduce the smooth case to the singular one, and how to handle varieties of dimension ≥ 3. The upshot is that the conjecture is false for every variety of dimension > 1. In a later paper he further proved that his motivic six-functor theory is compatible with the Betti realization—in summary: a substantial part of his research has been around the six-functor formalism, where he has done ground-breaking work with important implications.

Ayoub has also done other very important work, for example the paper using motivic methods to illuminate the algebro-geometric aspects of Borel-Serre compactifications of locally symmetric spaces. Perhaps most interesting are theorems comparing the motivic cohomology of sheaves on one compactification with the analytic cohomology of sheaves on another.

Citation—Moritz Kerz

Kerz has done several important pieces of work. One of his early theorems asserts the following: if R is a semilocal, regular connected ring with quotient field F, and all the residue fields are infinite, then the natural map in Milnor K–theory $K_n^M(R) \longrightarrow K_n^M(F)$ is universally injective for all $n \geq 0$. As applications he deduces a conjecture of Beilinson's on the motivic complex of Zariski sheaves $\mathbb{Z}(n)$, as well as a conjecture of Levine's generalizing the (proved) Bloch-Kato conjecture on the norm residue homomorphism.

There is the article with Saito, which proves Kato's conjectures for higher-dimensional arithmetic schemes. The conjectures assert the vanishing of the Kato cohomology with coefficients in $\mathbb{Z}/n\mathbb{Z}$, and they prove it assuming n is invertible. This is a sharp improvement on the known results, which were either in low dimension or assumed resolution of singularities. As applications they prove new finiteness results for motivic cohomology with finite coefficients, and obtain formulas for certain special values of zeta functions.

A recent article, with Bloch and Ésnaut, shows that the obstruction to lifting certain classes in K_0 to formal neighborhoods is purely cohomological. Precisely: let k be a perfect field of characteristic $p > 0$ and let $W = W(k)$ be the ring of Witt vectors over k. Let X be a smooth projective variety over W, and let X_1 be the special fiber. Then an element $\xi_1 \in K_0(X_1)_{\mathbb{Q}}$ lifts to all infinitesimal neighborhoods if and only if it belongs to the right part of the Hodge filtration on de Rham cohomology.